CapOptix: An Options-Approach based Framework for Capacity Market Pricing Models

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Introduction

With zero-carbon emission targets, i.e. addition of Renewable Energy to mix :

- Uncertainty in generation due to weather.
- Demand not fulfilled i.e. $Demand_t > Supply_t \implies$ Demand Shed and Power Outages. Therefore, leads to **Security of Supply (SoS) Problem.** [1, 4]

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Current Solution -

- Resources:
  1. Usage of Spinning Reserves (mostly conventional power plants)
  2. Storages etc.
- Demand Response Programs.
- Market's Point of View:
  1. Day-ahead Market Operation with Real-time Balancing.
  2. Existence of **Capacity Remuneration Mechanisms**

Introduction

What are CRMs?

All those policies whose aim is explicitly to remunerate capacity in order to **provide the proper level of generation adequacy**.

- Secure Investments in Generation Capacity
- Grid Operation
- Security and Reliability

Introduction

1. **Capacity Payments:** payments for capacity, administratively set. (eg: in Portugal, **Spain**, Poland, Argentina)
2. **Capacity Obligations:** hold enough capacity to serve load.(MISO, **CAISO**)
3. **Strategic Reserves:** power plants withdrawn from markets and divested to the SO, that uses them whenever there is a SoS threat.(Belgium, **Germany**)
4. **Capacity Auctions**: price discovery mechanism through which the SO remunerates a given amount of generation capacity (eg: PJM, **NYISO**)
   - **Reliability Options** (Colombia, ISO New England, Ireland and **Italy**)

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- Does it really solve the "missing money" problem?
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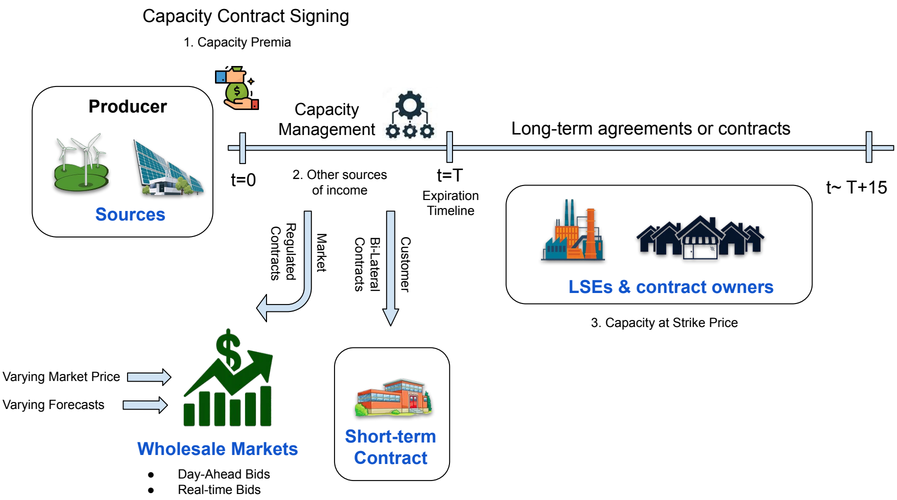
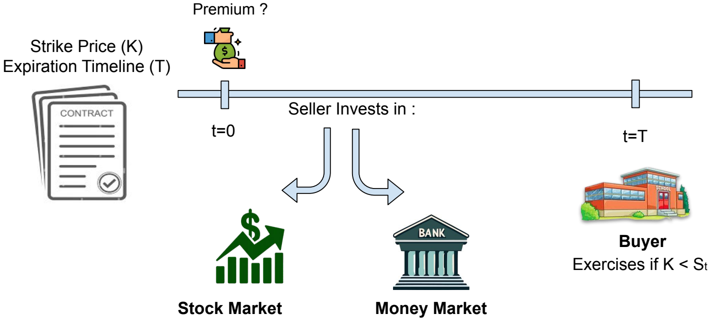
Our Goal:

Develop Capacity Market Pricing Models to figure out the Premia charged.

Reliability-based Capacity Market Working



Similarity with Finance: Option Theory



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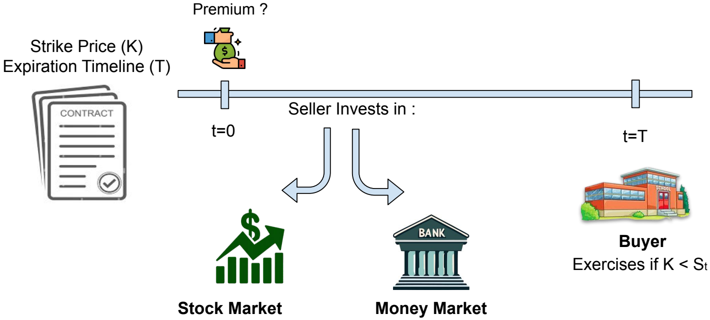


Theorem : Value of Call European Option at time T

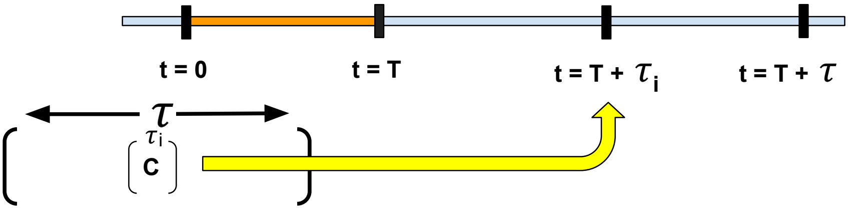
Worth of the portfolio at time T = Loss incurred by selling to buyer

$$C(T,(S(T)) = max\{S_T - K, 0\}$$

Premium = C(0,S(0))

Similarity with Finance: Option Theory

Capacity Markets :



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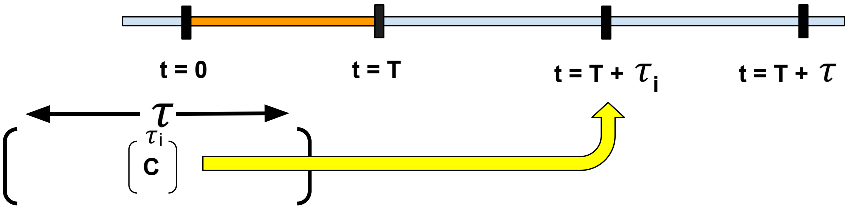

Theorem : Capacity Markets are a Series of European Option.

Premium charged:

$$C(0, S(0)) = \sum_{t=T}^{T+\tau} e^{-rt}\mathbb{E}[C(t, S_t)] = \sum_{t=T}^{T+\tau} e^{-rt}\mathbb{E}[max\{S_t - K, 0\}]$$

Our Case Study: Regions of Austria

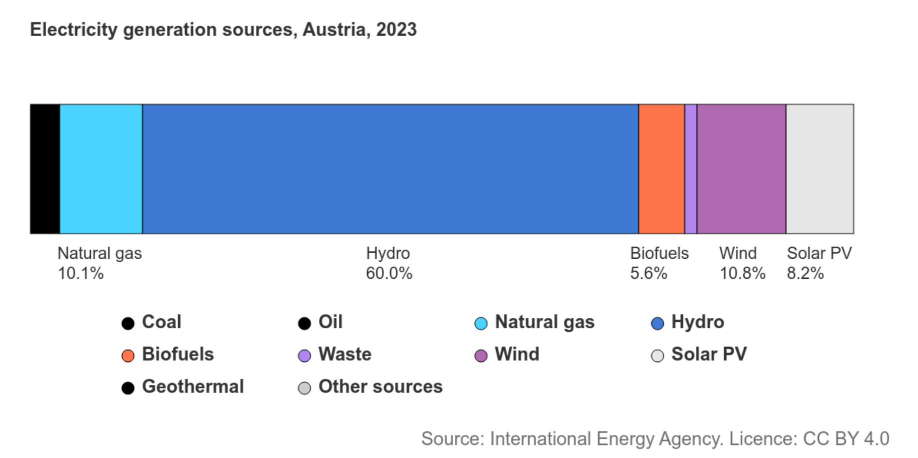

Figure: Total electricity production in Austria : 74,151 GWh

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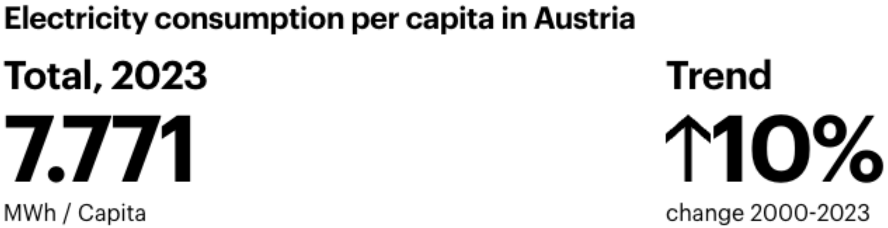
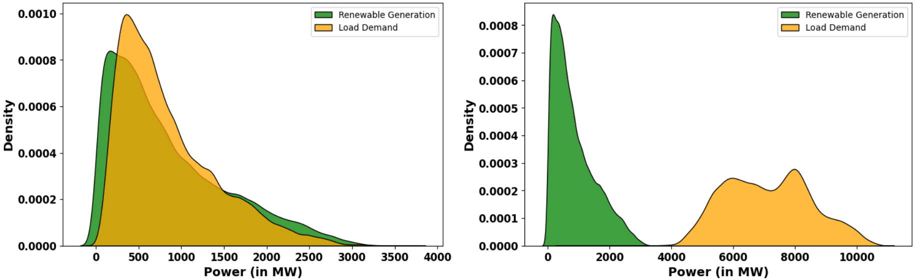


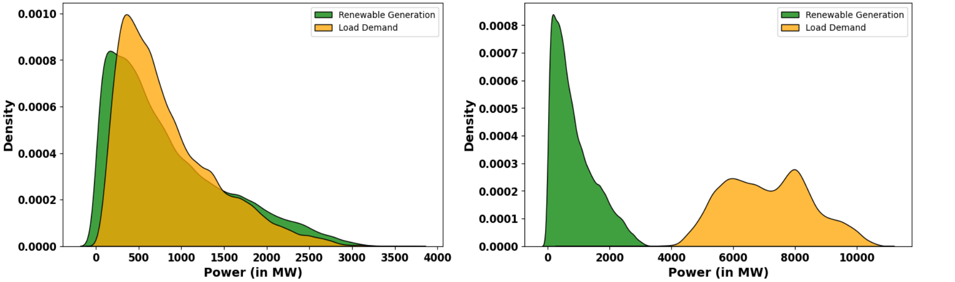
Figure: Total electricity consumption in Austria : 7.771 MWh/capita $\times$9.13 million $\approx$ 70,950 GWh



(a) Region A (b) Region B

Figure: Distribution of Supply-Demand Mismatches

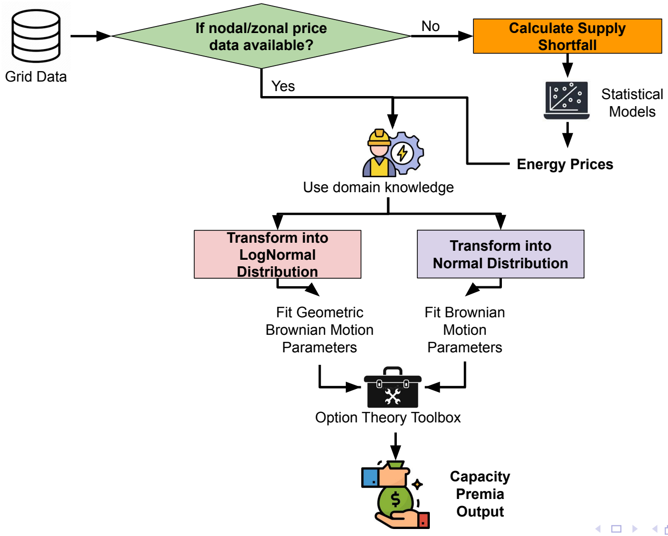
Our Case Study: Supply-Demand Distribution



(a) Region A (b) Region B

Figure: Distribution of Supply-Demand Mismatches

Region A has a mature Renewable Energy Supply.
Region B needs infrastructural development (Assume: no imports allowed).



Grid Data
If nodal/zonal price data available?
No
Yes
Calculate Supply Shortfall
Statistical Models
Energy Prices
Use domain knowledge
Transform into LogNormal Distribution
Transform into Normal Distribution
Fit Geometric Brownian Motion Parameters
Fit Brownian Motion Parameters
Option Theory Toolbox
Capacity Premia Output

CapOptix: Price Distribution

In Finance, Stock Prices are modelled as Geometric Brownian Motion:

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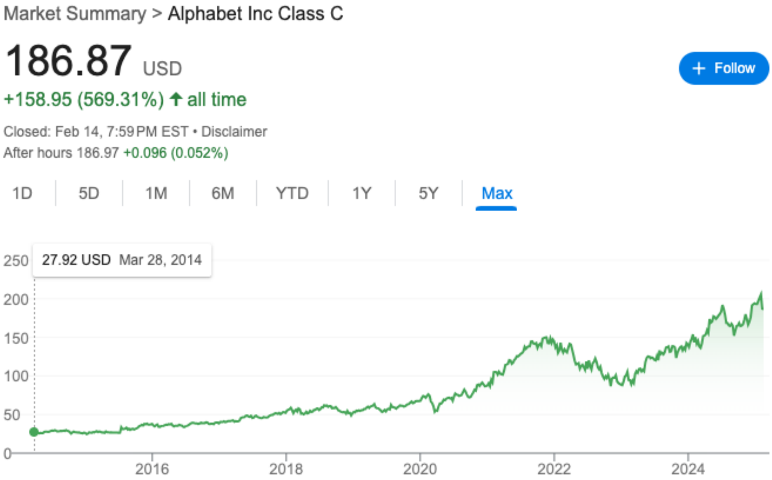

CapOptix: Price Distribution - Austria

Does Geometric Brownian Motion always model Energy Prices? : No!

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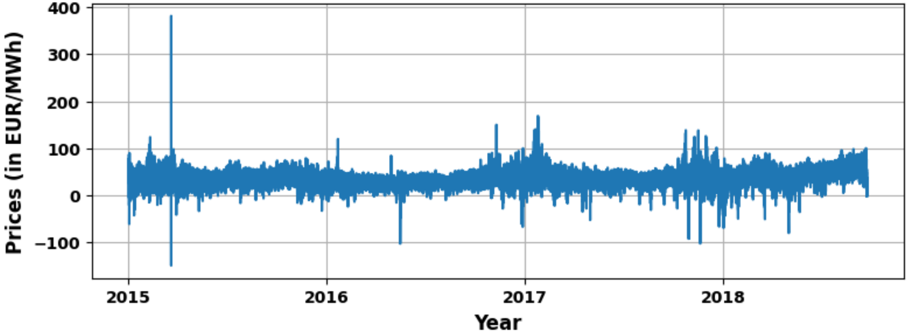

Figure: Day-Ahead Energy Price Variation across Years [3]

CapOptix: Price Distribution - Austria

We use the Ornstein-Uhlenbeck (OU) process by verifying the evolution of a value S_t over time, which is governed by the stochastic differential equation (SDE):

$$dS_t = \theta(\mu - S_t)dt + \sigma dW_t$$

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Next, we introduce jumps by adding Poisson jump process. The model results in :

$$dS_t = \theta(\mu - S_t)dt + \sigma dW_t + J_t$$

where, $J_t = Y.N_t$:

1. $Y \sim \mathcal{N}(\mu_J, \sigma_j^2)$: Jump size distribution
2. $N_t \sim Poisson(\lambda)$: counts the number of jumps in a time interval, with λ as the expected number of jumps per unit time.

After simulating the above process (by fine-tuning parameters):

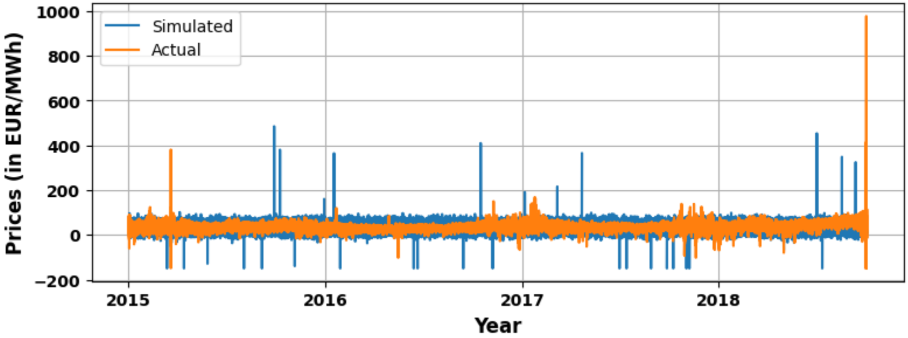

Figure: Day-Ahead Energy Price Variation across Years

When Nodal information of prices not present :-

1. Calculate Supply Shortfall.
2. Use MLE to fit supply shortfall to regional energy prices.
3. Outcome -Processed Energy Prices which are used as underlying for the Capacity Market Premium Determination.

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Standard Brownian Motion models Region A, since it intermittently falls short of the demand and at times can produce excess as well.

Geometric Brownian Motion models Region B since there is a huge shortfall.

Model A - GBM without jump : $\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$ for Region B

$$C(0, S_0) = \sum_{t=T}^{T+\tau} S_0 \Phi(d_2 + \sigma\sqrt{t}) - e^{-rt} K \Phi(d_2)$$

where $d_2 = -\frac{\log\left(\frac{S_0}{K}\right) + \left(r - \frac{1}{2}\sigma^2\right)t}{\sigma\sqrt{t}}$ [2]

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Model B - BM without jumps : $dS_t = \mu S_t dt + \sigma dW_t$ for Region A

$$C(0, S_0) = \sum_{t=T}^{T+\tau} e^{-rt} m\phi(n) + e^{-rt}(S_0 e^{rt} - K)\Phi(n)$$

where n= $\frac{S_0 e^{rt} - K}{m}$ and m $= \sqrt{\frac{\sigma^2}{2r}(e^{2rt} - 1)}$ [2]

For Region B (Model C) -

Model C - GBM with jumps : $\frac{dS_t}{S_t} = \mu dt + \sigma dW_t + Jumps$

$$C(S_0^{(n)}, 0) = \sum_{t=T}^{T+\tau} \sum_{n=0}^{\infty} C(S_0^{(n)}, 0 | N_t = n) \frac{(\lambda t)^n e^{-t}}{n!}$$

where $\sigma_n = \sqrt{\sigma^2 + \frac{n\sigma_y^2}{t}}$, $S_0^{(n)} = S_0 e^{(n\mu_y + \frac{n\sigma_y^2}{2})}$ [5]

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For Region A (Model D)-

1. Model Jumps similar to Model C except that Jump size is sampled from Normal Distribution.
2. When there are more jumps in small interval dt, becomes a product of random variables following Normal Distribution.
3. **No Closed-form** Solution exists.
4. Use Monte-Carlo to evaluate the discounted expected value of the capacity:

$$C(0, S_0) = \sum_{t=T}^{T+\tau} e^{-rt} \mathbb{E}^{\mathbb{Q}}[\{S_t - K\}^+]$$

Results: Capacity Premia

Sl.	Region	Underlying Energy Price	Model Specifications	Capacity Premia (€/MWmonth)	Strike Price (€/MWh)
1	A	Derived Zonal Energy Price from Shortfall	Model B	14,402 ∼20 €/MWh	33
2	B	Derived Zonal Energy Price from Shortfall	Model A	91,340 ∼126.86 €/MWh	31
3	∼	Central Price Data	Model D	30,520 ∼42.38 €/MWh	36
4	∼	Central Price Data	Model C	25,471 ∼35.37 €/MWh	36

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- These are capacity premia over 2 years until the expiration timeline.
- Then, a strike price is charged for the contract period of 10 years.

Results: Austria

Sl.	Region	Capacity Premia (€/MWmonth)	Strike Price (€/MWh)	Energy Price Stats (€/MWh)
1	Supply $\sim$Demand	14402 $\sim$20 €/MWh	31	Mean : 31 std-dev :1.76
2	Supply $<<$Demand	91,340 126.86 €/MWh	31	Mean : 31 std-dev :6.4
3	Supply $\sim$Demand	0	125	same as Sl. 1
4	Supply $<<$Demand	87021 $\sim$120.86 €/MWh	125	same as Sl. 2

Results: Italy

Sl.	Time	Capacity Premia (€/MWmonth)	Strike Price (€/MWh)	Energy Price Stats (€/MWh)
1	Jan 2009 - Dec 2010	5.4k 7.49 €/MWh	125	Mean : 63.9 std-dev :2.69
2	Jan 2016 - Dec 2017	0.025k ∼0.034 €/MWh	125	Mean : 48 std-dev :2.97
3	Jan 2022 - Dec 2023	971k ∼1348 €/MWh	125	Mean : 215.61 std-dev :56.49

Ongoing Work and Future Scope

1. Theoretically prove that Reliability Options increase reliability by reducing loss of load probability i.e. $\mathbb{P}(Demand_t > Supply_t)$.
2. Contract Length Time seems to be a very important factor for premium calculation. How to determine? Ans: Through Cost Recovery!
3. Price Distribution of various Regions follow different process (CAISO and NYISO similar process as Austria). Hence the Option Theory Toolbox needs to expand! Example in next slide:-

Ongoing Work and Future Scope

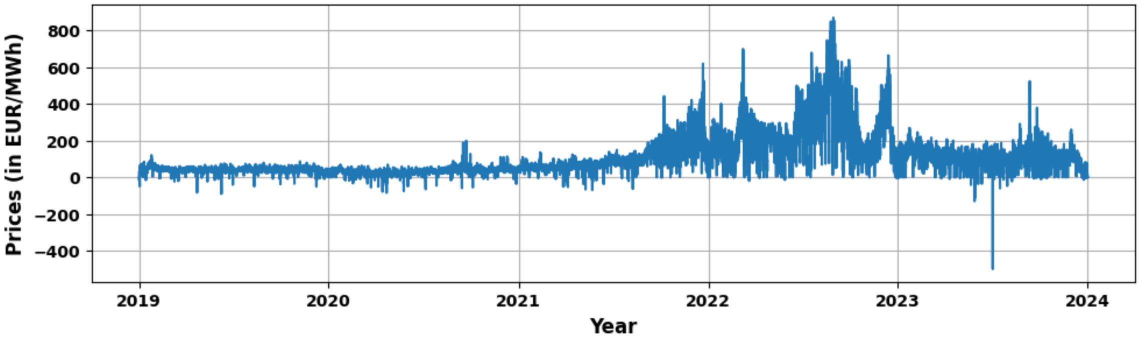

Figure: Energy Prices in Germany, *SOURCE: SMARD - Strommarktdaten*.

Ongoing Work and Future Scope

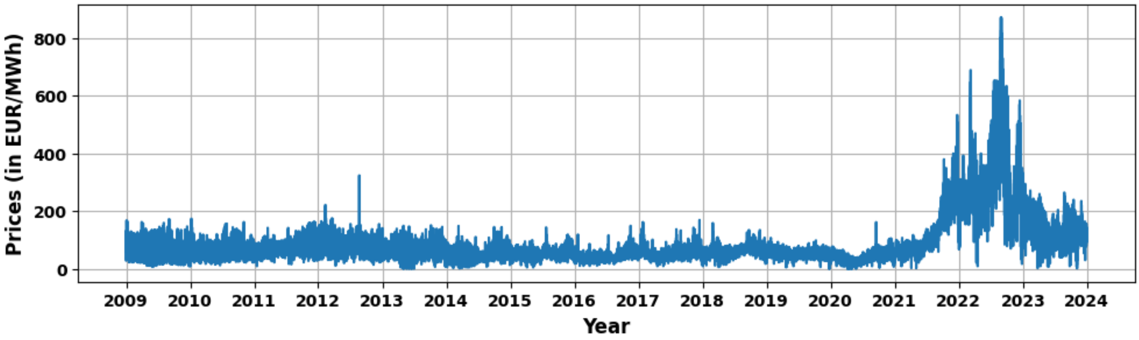

Figure: Energy Prices in Italy, *SOURCE: Terna - Italian Electricity Transmission Operator*.

Ongoing Work and Future Scope

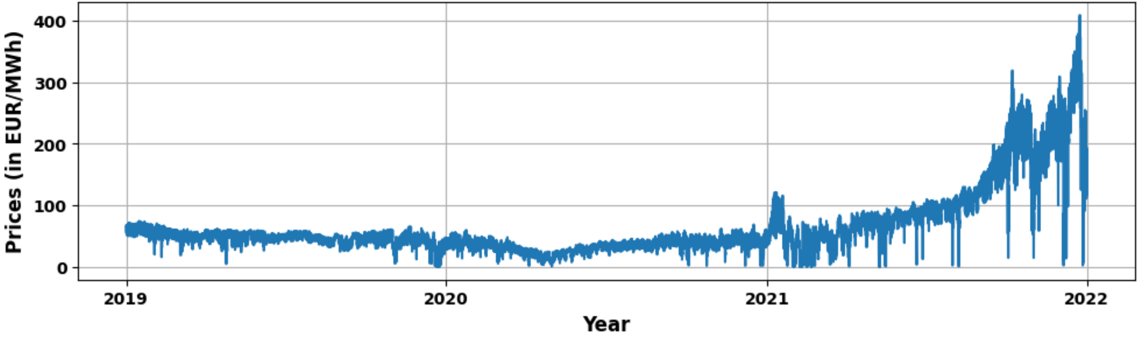

Figure: Energy Prices in Spain, *SOURCE: OMIE - Iberian Electricity Market Operator*.

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Thank You!